Deployment Optimization of Uniform Linear Antenna Arrays for a Two-Path Millimeter Wave Communication System

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Abstract—This letter is concerned with the array design of a fixed point-to-point millimeter wave (MMW) communication system consisting of two paths between the transmitter and receiver. We assume uniform linear antenna arrays (ULAs) at both the transmitter and receiver and focus on the optimization of antenna deployment to maximize the system capacity with uniform power allocation. We analytically show that, at high signal-to-noise ratios (SNRs), the channel capacity depends on the spatial correlations between the two paths at both link ends. The optimal antenna array deployment that maximizes the channel capacity is derived. Numerical results show that the optimized antenna array deployment considerably improves the channel capacity, compared to the non-optimized deployment.

Index Terms—Millimeter wave (MMW) communications, uniform linear antenna array (ULA), antenna deployment optimization.

I. INTRODUCTION

With the growing demand for ultra-high data rate transmissions [1], millimeter wave (MMW) communications have attracted considerable attentions to achieve multi-gigabit data rate transmission due to the large available spectrum at MMW frequency band [2]. However, MMW signal transmissions experience severe propagation loss due to the extremely high operating frequencies. Fortunately, the small wavelengths at MMW enable the integration of a large number of antennas to compensate the severe path loss. Some ultra-high data rate MMW multiple-input multiple-output (MIMO) systems have been demonstrated [1]–[3].

Different from the conventional microwave systems that are generally modeled to be rich scattering, an MMW MIMO system is featured by its highly directive transmission and much reduced scattering effect [2]. In particular, when the transmitter and receiver are in the visible ranges of each other, the channel is mainly dominated by line-of-sight (LoS) transmissions. In this case, the corresponding pure LoS channel behavior has been investigated in [4], [5]. However, recent experiments [6] show that in a practical MMW communication environment such as urban areas, the surrounding obstacles (e.g., buildings) can likely lead to a few reflection paths with non-ignorable channel energy between the transmitter and receiver. In [7], [8], the MMW MIMO systems with both LoS and reflected paths have been considered, and the effect of propagation environment on the channel capacity has been investigated via experiments. It is shown that the antenna deployment (i.e., antenna spacing and antenna array orientation) can significantly affect the system performance. The optimal antenna deployment design for a 2 × 2 MIMO system was proposed in [9] to enhance the channel capacity, but this work cannot be extended to a general case with more antenna elements.

In this letter, we focus on the antenna orientation design in a two-path MMW MIMO system with uniform linear antenna arrays (ULAs). We first investigate the relationship between the channel capacity and the spatial correlation at high signal-to-noise ratios (SNRs). Based on this relationship, we analytically derive the optimal orientations of both the transmit and receive ULAs. Numerical results show that a considerable improvement of the channel capacity can be achieved after optimization.

II. SYSTEM MODEL

Consider a point-to-point MMW MIMO communication system with an $N_t$-element ULA at the transmitter and an $N_r$-element ULA at the receiver. There are two main distinguishable paths between the transmitter and the receiver, and the fading coefficients for both the two paths are comparative. To facilitate understanding, we assume the special cases that one of them is the LoS path, and the other is a first-order reflected path, as illustrated in Fig. 1.

Specifically, the transmit ULA is centered on the negative half of the x-axis with distance $D/2$ from the origin and angle $\theta_{tL}$ relative to the positive half of the x-axis. The receive ULA is centered on the positive half of the x-axis with the distance $D/2$ from the origin and angle $\theta_{rL}$ relative to the negative half.

Fig. 1. 2D geometrical model of ULA MIMO system with LoS and reflected paths.
of the $x$-axis. It is worth noting that, physically, $\theta_{tL}$ and $\theta_{rL}$ are also, respectively, the angle of departure (AoD) and angle of arrival (AoA) of the LoS path relative to the transmit and receive ULAs.

Without loss of generality, we assume that the reflecting point, marked by $C(x_c, y_c)$ in Fig. 1, is in the first quadrant. Denote by $\theta_{tR}$ and $\theta_{rR},$ respectively, the AoD and AoA of the reflected path relative to the transmit and receive ULAs. Then the angle differences between the LoS and reflected paths at the transmitter and receiver can be written as $\Delta \theta_t = \theta_{tL} - \theta_{tR}$ and $\Delta \theta_r = \theta_{rL} - \theta_{rR},$ respectively. In this letter, we assume that the angle differences $\Delta \theta_t$ and $\Delta \theta_r$ are fixed and that $\theta_{tL}$ and $\theta_{rL}$ are within the ranges $[\Delta \theta_t/2, \Delta \theta_t/2 + \pi]$ and $[\Delta \theta_r/2, \Delta \theta_r/2 + \pi],$ respectively. Our focus is on the optimization of antenna orientations, i.e., $\theta_{tL}$ and $\theta_{rL},$ for capacity maximization.

Following [10], here we focus on a far-field distance between the transmit and receive ULAs. The overall channel matrix, denoted by $H,$ can be expressed as the sum of two rank-one matrices, corresponding to the LoS path and reflected path respectively. Mathematically, we have

$$H = a_L e_r(\theta_{rL}) e_l(\theta_{tL})^H + a_R e_r(\theta_{rR}) e_l(\theta_{tR})^H,$$

where $a_L$ and $a_R$ are, respectively, the fading coefficients for the LoS path and the reflected path. For convenience, we normalize $a_L = 1.$ Then according to the free space propagation law, $a_R$ can be expressed as

$$a_R = |a_R|e^{j\Delta \theta} = \sigma D e^{-j2\pi(D'-D)\theta},$$

where $|\cdot|$ denotes the absolute value operator, $D'$ is the length of the reflected path, $e^{j2\Delta \theta}$ is the phase difference between the two paths, and $\sigma$ is the complex reflection coefficient which denotes the power loss and phase difference caused by absorption and scattering at the reflecting point $C(x_c, y_c).$ In (1), $e_r(\theta_r)$ and $e_l(\theta_r)$ are, respectively, the receive and transmit spatial signatures for a path with AoA $\theta_r$ and AoD $\theta_r$. They are defined as

$$e_r(\theta_r) = \frac{1}{\sqrt{N_r}} \begin{bmatrix} e^{-j2\pi d_r \cos \theta_r} \\ \vdots \\ e^{-j2\pi(N_r-1)d_r \cos \theta_r} \end{bmatrix},$$

and

$$e_l(\theta_l) = \frac{1}{\sqrt{N_t}} \begin{bmatrix} e^{-j2\pi d_t \cos \theta_l} \\ \vdots \\ e^{-j2\pi(N_t-1)d_t \cos \theta_l} \end{bmatrix},$$

where $d_t$ and $d_r$ are, respectively, the transmit and receive antenna spacings measured in terms of the numbers of wavelength.

III. CAPACITY ANALYSIS AND ANTENNA DESIGN

According to (1), the channel matrix $H$ is of rank 2. Denote by $\lambda_1$ and $\lambda_2$ the eigenvalues of $HH^H.$ The channel capacity with uniform power allocation$^2$ on both eigenchannels can be calculated as

$$C = \log_2 \left( \det \left( I + \frac{1}{2} \rho HH^H \right) \right)$$

$$= \log_2 \left( \frac{1}{2} e^{2\lambda_1} + e^{2\lambda_2} \right)$$

$$= \log_2 \left( 1 + \rho \lambda_1 \lambda_2 + \frac{1}{2} \rho^2 \lambda_1^2 \lambda_2^2 \right),$$

where $\rho$ is the transmit SNR. It can be seen that the capacity depends on the values of the two eigenvalues $\lambda_1$ and $\lambda_2,$ which are in turn determined by the antenna orientations $\theta_{tL}$ and $\theta_{rL}.$ In this section, we will first analyze the impact of $\theta_{tL}$ and $\theta_{rL}$ on the channel capacity. The optimal orientations of both the transmit and receive ULAs will then be provided based on our analysis.

A. Capacity Analysis

To facilitate our discussion below, we rewrite the channel matrix as

$$H = \left[ e_r(\theta_{rL}) e_l(\theta_{tL}) \right] + \left[ e_r(\theta_{rR}) e_l(\theta_{tR}) \right] =\begin{bmatrix} H_{\text{LoS}} & H_{\text{Reflected Path}} \end{bmatrix},$$

Performing QR decomposition on the first multiplicative matrix of (6), we obtain

$$H_{\text{LoS}} = e_r(\theta_{rL}) e_l(\theta_{tL}) = Q_r R_r,$$

where $Q_r$ is a $N_r \times 2$ unitary matrix, and

$$R_r = \begin{bmatrix} f_r(\theta_{rL}) e^{j\Delta \theta} & 0 \\ 0 & \sqrt{1 - |f_r(\theta_{rL})|^2} \end{bmatrix},$$

where $f_r(\theta_{rL}) = e_r(\theta_{rL})^H e_r(\theta_{rR}) = e_r(\theta_{rL})^H e_r(\theta_{rL} - \Delta \theta_r)$ represents the spatial correlation between the LoS path and the reflected path at the receiver. Similarly, the QR decomposition of the third multiplicative matrix in (6) is given by

$$H_{\text{Reflected Path}} = e_r(\theta_{rL}) e_l(\theta_{tR}) = Q_r R_t,$$

where $Q_t$ is an $N_t \times 2$ unitary matrix, and

$$R_t = \begin{bmatrix} f_t(\theta_{tL}) e^{j\Delta \theta} & 0 \\ 0 & \sqrt{1 - |f_t(\theta_{tL})|^2} \end{bmatrix},$$

where $f_t(\theta_{tL}) = e_t(\theta_{tL})^H e_t(\theta_{tR}) = e_t(\theta_{tL})^H e_t(\theta_{tL} - \Delta \theta_l)$ is the spatial correlation at the transmitter.

Substituting (7) and (9) into (6), we can rewrite $H$ as

$$H = Q_r R_r \begin{bmatrix} 1 & |a_R| \end{bmatrix} R_t^H Q_t^H = Q_r H Q_t^H,$$

where $H = R_r \begin{bmatrix} 1 & |a_R| \end{bmatrix} R_t^H,$ whose detailed expression is given in (12) on the top of next page. Since $Q_r$ and $Q_t$ are both reduced unitary matrices, $H$ has the same singular values as the channel matrix $H.$ Consequently, $\lambda_1$ and $\lambda_2$ are also the

$^1$This assumption facilitates our discussion below, and it is equivalent to assume that the ranges of $\theta_{tL}$ and $\theta_{rL}$ are both $[0, \pi].$

$^2$Since the optimal power allocation using water-filling cannot be treated analytically in this paper we consider the equal power allocation. The assumption of equal power is very close to reality, as it can be validated by simulations that the gap between the water-filling capacity and that with uniform power allocation is low (less than 0.6 bit/s/Hz) for typical SNRs (3-20dB).
The resultant capacity loss is marginal (less than 0.5 bit/s/Hz) for low SNRs via exhaustive search that it is also near-optimal in the case of finite SNRs.

Therefore, by substituting (16) into (5), we can have the approximation of the channel capacity at high SNRs as

$$\lim_{\rho \to \infty} C \approx \log_2 (p^2 \lambda_1 \lambda_2 / 4)$$

or equivalently

$$\lambda^2 - (g_{11} + g_{22}) \lambda + g_{11}g_{22} - g_{12}g_{21} = 0.$$  \hspace{1cm} (15)

Then the following relationship holds for $\lambda_1$ and $\lambda_2$.

$$\lambda_1 \lambda_2 = g_{11}g_{22} - g_{12}g_{21} = |a_R|^2 (1 - |f_r(\theta_{tL})|^2)(1 - |f_r(\theta_{tL})|^2).$$  \hspace{1cm} (16)

Therefore, by substituting (16) into (5), we can have the approximation of the channel capacity at high SNRs as

$$\lim_{\rho \to \infty} C \approx \log_2 (p^2 \lambda_1 \lambda_2 / 4)$$

Since both $|f_r(\theta_{tL})|$ and $|f_r(\theta_{tL})|$ are independent variables ranging from 0 to 1, the channel capacity in (17) is maximized when both $|f_r(\theta_{tL})|$ and $|f_r(\theta_{tL})|$ are minimized individually. Thus, the maximum channel capacity at high SNRs can be achieved by solving the following two optimization problems

$$\min |f_r(\theta_{tL})| \quad \text{s.t.} \quad \theta_{tL} \in [\Delta \theta / 2, \Delta \theta / 2 + \pi]$$

and

$$\min |f_r(\theta_{tL})| \quad \text{s.t.} \quad \theta_{tL} \in [\Delta \theta / 2, \Delta \theta / 2 + \pi].$$

Since problems (18) and (19) are very similar, in the following subsection, our discussion will be mainly confined to (18).

**B. Optimal Antenna Deployment Design**

The optimal solution to problem (18) is presented in the following proposition.

**Proposition 1**: The minimum value of the spatial correlation at the receiver is given by

$$|f_r(\theta_{tL})|_{\min} = \begin{cases} 0, & \Delta \theta \geq 2 \arcsin \frac{1}{2N_r d_r}, \\ \sin(2\pi N_r d_r \sin \Delta \theta / 2) / N_r \sin(2\pi d_r \sin \Delta \theta / 2), & \Delta \theta < 2 \arcsin \frac{1}{2N_r d_r}, \end{cases}$$

which is achievable when the receive ULA orientation satisfies

$$\theta_{tL} = \begin{cases} \arcsin(\Delta \phi_r) + \Delta \theta / 2, & \Delta \theta \geq 2 \arcsin \frac{1}{2N_r d_r}, \\ \pi / 2 + \Delta \theta / 2, & \Delta \theta < 2 \arcsin \frac{1}{2N_r d_r}. \end{cases}$$

Modulo (·, ·) denotes the modulo operator.

**Proof**: The spatial correlation at the receiver can be derived as

$$|f_r(\theta_{tL})| = |e_r(\theta_{tL})| \frac{1}{\sqrt{\pi N_r d_r}}$$

where $\Delta \phi_r = 2 \pi \frac{N_r d_r \sin \Delta \theta_1}{2}$, $p = 1, 2, \ldots, \lfloor 2N_r d_r / \Delta \theta_1 \rfloor$, $\theta_{tL} \in [\frac{\pi}{2} + \Delta \theta / 2, \pi]$.

Since $\theta_{tL} \in [\Delta \theta / 2, \Delta \theta / 2 + \pi]$, (23) holds only when the right-hand-side also falls in the interval $[0, 1]$. This requires

$$\Delta \theta \geq 2 \arcsin \frac{1}{2N_r d_r}.$$  \hspace{1cm} (26)

Thus we have

$$\sin \frac{2\theta_{tL} - \Delta \theta}{2} = \frac{p}{2N_r d_r \sin(\Delta \theta / 2)}.$$  \hspace{1cm} (25)

Since the left-hand-side of (25) can only take values from $[0, 1]$ when $\theta_{tL} \in [\Delta \theta / 2, \Delta \theta / 2 + \pi]$, (23) holds only when the right-hand-side also falls in the interval $[0, 1]$. This requires

$$\Delta \theta < 2 \arcsin \frac{1}{2N_r d_r}.$$  \hspace{1cm} (28)

Similarly, the solution to problem (19) is presented in Proposition 2 below. Its proof is omitted here for brevity.
Proposition 2: The minimum value of the spatial correlation at the transmitter is given by
\[
|f_t(\theta_{tL})|\min = \begin{cases} 
0, & \Delta \theta_t \geq 2\arcsin \frac{1}{2N_r d_t}, \\
\sin(2\pi N_r d_t \sin \frac{\Delta \theta_t}{2N_r d_t}), & \Delta \theta_t < 2\arcsin \frac{1}{2N_r d_t}, 
\end{cases}
\]
which is achievable when the transmit ULA orientation satisfies
\[
\theta_{tL} = \begin{cases} \arcsin(\Delta \phi_t) + \Delta \theta_t/2, & \Delta \theta_t \geq 2\arcsin \frac{1}{2N_r d_t}, \\
\pi/2 + \Delta \theta_t/2, & \Delta \theta_t < 2\arcsin \frac{1}{2N_r d_t}, 
\end{cases}
\]
where \(\Delta \phi_t = \frac{p}{2N_r d_t \sin \frac{\Delta \theta_t}{2}}, p = 1, 2, ..., \lfloor 2N_r d_t \sin \frac{\Delta \theta_t}{2} \rfloor, \mod (p, N_r) \neq 0.\)

In summary, by setting the receive and transmit ULA orientations according to (21) and (32), both the receive and transmit spatial correlations \(|f_r(\theta_{rL})|\) and \(|f_t(\theta_{tL})|\) can achieve their minimum values as (20) and (31), respectively. Correspondingly, the considered MIMO channel capacity is maximized.

IV. Numerical Results

In this section, we consider a practical MMW MIMO system with \(N_r = N_t = 6\) and \(D = 100\) meters. Both the transmit and receive antenna elements are assumed to be half-wavelength spaced, i.e., \(d_t = d_r = 0.5\). When calculating the fading coefficient \(a_{br}\) based on (2), we set the reflection power loss equal to be -3dB. The transmit SNR is fixed at \(p = 20dB\).

The channel capacity versus the location of the reflecting point is shown in Fig. 2. From the figure we can see that the channel capacity not only relates to the angle differences at the receiver and the transmitter, but also relates to the length of the reflected path. On one hand, larger angle differences, e.g., \(\Delta \theta_t \geq 2\arcsin \frac{1}{2N_r d_t}\) and \(\Delta \theta_t \geq 2\arcsin \frac{1}{2N_r d_t}\) as indicated by Propositions 1 and 2, can potentially provide two orthogonal paths at both sides, leading to a significantly higher channel capacity. On the other hand, a larger reflected path length involves more severe path loss and so degrades the channel capacity. Combining these two effects, we can see that the highest channel capacity concentrates in the region near the intersection of the two lines that correspond to the thresholds of the transmit and receive angle differences, i.e., \(\Delta \theta_t = 2\arcsin \frac{1}{2N_r d_t}\) and \(\Delta \theta_t = 2\arcsin \frac{1}{2N_r d_t}\). It is also seen that when the reflecting point is very close to the communication link, even much less propagation loss is involved in the reflected path, the achievable channel capacity still degrades a lot. This is because the two paths are highly correlated in this case, leading to an approximate rank-one channel matrix.

Table I lists the channel capacities of the considered system when the reflecting points are randomly located at the 5 positions marked by white pentagrams in Fig. 2. For each location of the reflecting point, we consider the following three settings with both two ULABs are deployed: (1) at optimal orientations; (2) at worst orientations (the corresponding channel matrix becomes rank-one); (3) at random orientations in the ranges defined in (18) and (19) (the corresponding channel capacities in the table are averaged over the distribution of ULA orientations). From Table I we can see that, compared to arbitrary ULA orientations, optimal antenna deployment considerable improves the channel capacity by about 10 ~ 18%, and the potential improvement of channel capacity from the worst cases can even reach about 48 ~ 208%.

TABLE I

<table>
<thead>
<tr>
<th>C(xr, yr)</th>
<th>(−20, 30)</th>
<th>(−30, 30)</th>
<th>(5, 15)</th>
<th>(35, 10)</th>
<th>(−40, 15)</th>
</tr>
</thead>
<tbody>
<tr>
<td>opt</td>
<td>11.850</td>
<td>11.782</td>
<td>12.119</td>
<td>10.748</td>
<td>11.322</td>
</tr>
<tr>
<td>worst</td>
<td>7.993</td>
<td>7.804</td>
<td>7.958</td>
<td>6.990</td>
<td>6.700</td>
</tr>
<tr>
<td>opt/ave</td>
<td>110.0%</td>
<td>110.2%</td>
<td>115.2%</td>
<td>115.3%</td>
<td>114.5%</td>
</tr>
<tr>
<td>opt/worst</td>
<td>145.4%</td>
<td>151.0%</td>
<td>171.7%</td>
<td>168.0%</td>
<td>169.0%</td>
</tr>
</tbody>
</table>

V. Conclusion

In this letter, we analyzed the impact of antenna array orientations on the capacity of a two-path MMW MIMO channel. The optimal antenna orientations have been analytically derived. Numerical results demonstrate a considerable capacity improvement after antenna orientation optimizations.

REFERENCES