MCALIB – A Tool for Automated Error Analysis

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INTRODUCTION

- Floating point arithmetic susceptible to rounding errors - loss of accuracy.
- Dynamic error analysis an effective tool for measuring sensitivity.
- Implementation difficult - requires significant modifications to existing source code.

AIMS

- Automate the quantitative analysis of floating point rounding errors.
- Improved methods for the analysis of results.
- Demonstrate applications.

BACKGROUND

Floating Point Arithmetic:

- Binary IEEE754 Floating Point - subset of real numbers:

$$x = (-1)^s m \beta^e$$

- Implemented as finite precision rounded arithmetic system. Exact values are rounded approximations of inexact values:

$$\begin{aligned}\hat{x} &= \mathbb{F}(x)\\ &= x(1+\delta)\end{aligned}$$

- Normalization stage can lead to cancellation

Monte Carlo Arithmetic:

- Track information lost - model inexactness using random perturbations:

$$\begin{aligned}inexact(x,t,\xi) &= x + 2^{e_x - t}\xi\\ &= (-1)^{s_x}(m_x + 2^{-t}\xi)2^{e_x}\end{aligned}$$

- Statistics on rounding error obtained through repeated computations
- Uniformly distributed random variable used for ξ
- Individual operators performed in terms of the inexact function:

$$x \circ y = round(inexact(inexact(x) \circ inexact(y)))$$

LIBRARY IMPLEMENTATION

- Operations converted with source to source compiler CILLY
- Generated file compiled with MCA library to produce executable:

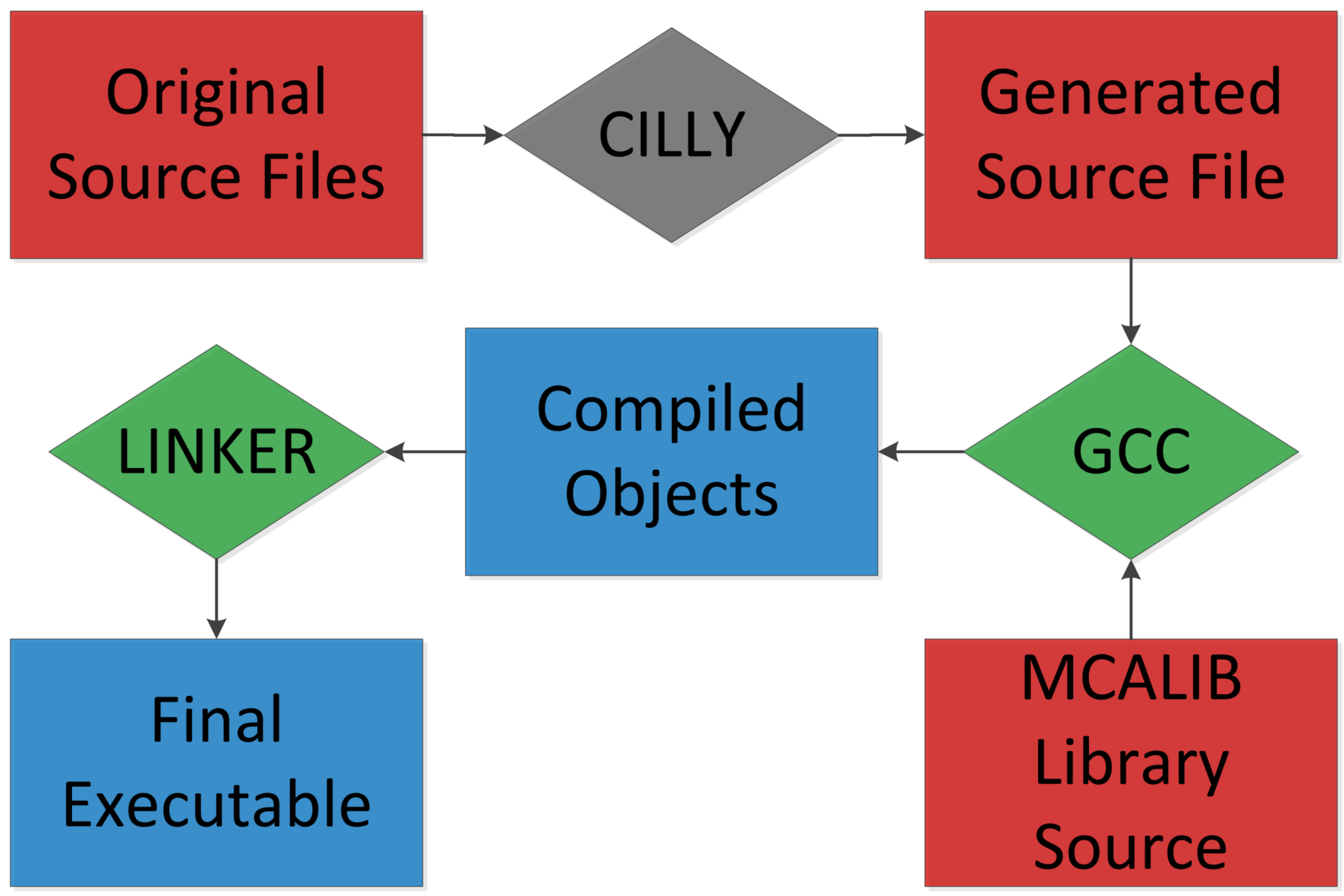

Figure 1: Source to Source compilation and Library implementation

METHODS FOR ANALYSIS OF RESULTS

- Results analysis in previous publications limited
- Approach may be more formally defined - provide better interpretation of MCA results
- Approach defines sensitivity to rounding error with two measurements:
  - The number of base-2 digits lost to rounding error, K
  - The minimum precision required to avoid a complete loss of significance, t_{min}
- Measurements are found by performing linear regression using relative standard deviation, (Θ), as the dependant variable and, *t*, as the exploratory variable:

$$\begin{aligned}\log_{10}(\Theta) &= \log_{10}(2^{K-t})\\ &= -\log_{10}(2)t + \log_{10}(2)K\\ &= mt + c\end{aligned}$$

- Robust regression methods are used to detect outlying data points
- Slope is a known variable - reduces problem to a 1D optimization

TESTING & RESULTS

- Statistical results and new analysis techniques allow for the comparison of algorithms and detection of catastrophic cancellation:

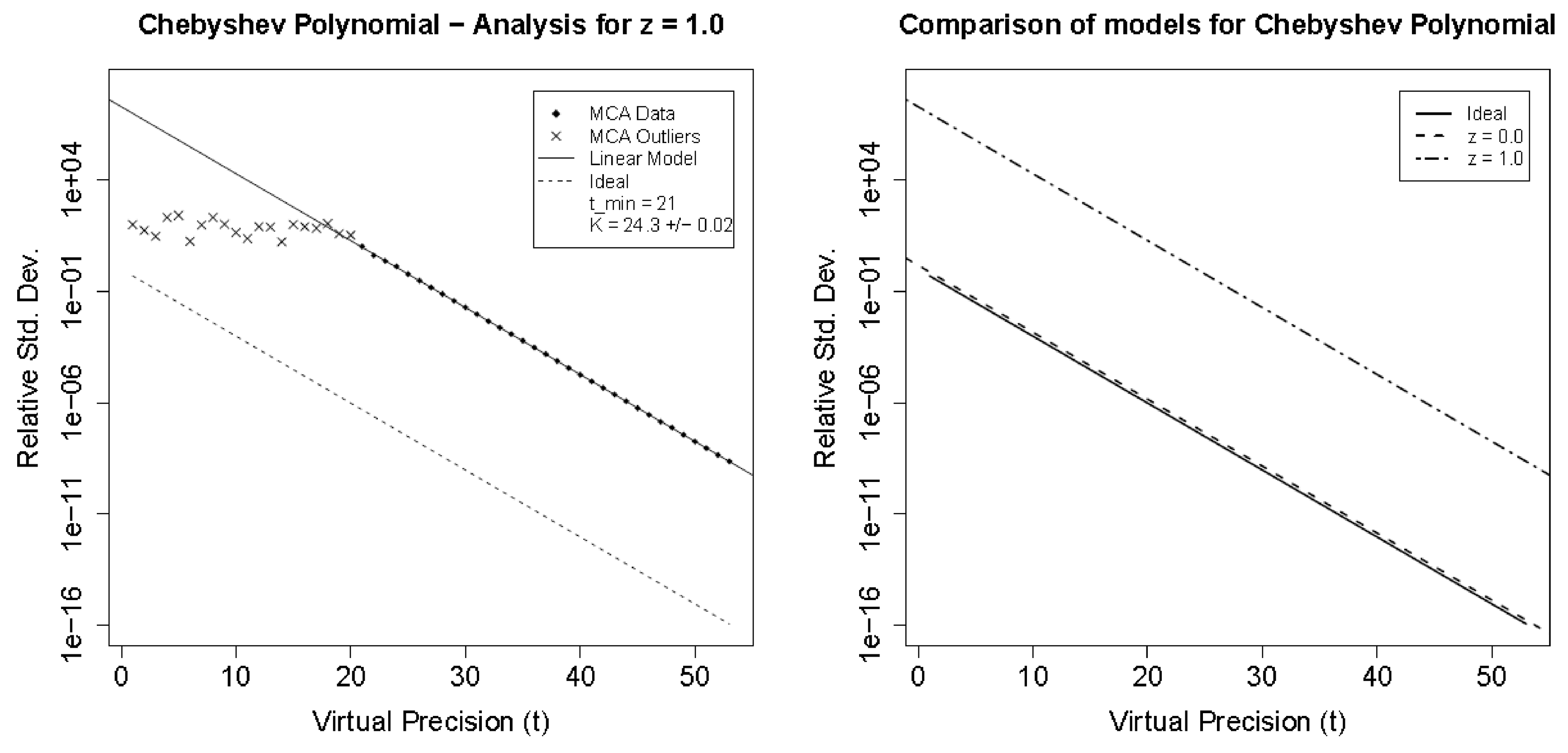

Figure 2: Comparison of results for Chebyshev Polynomial

- Analysis of results allows for optimization of algorithms:

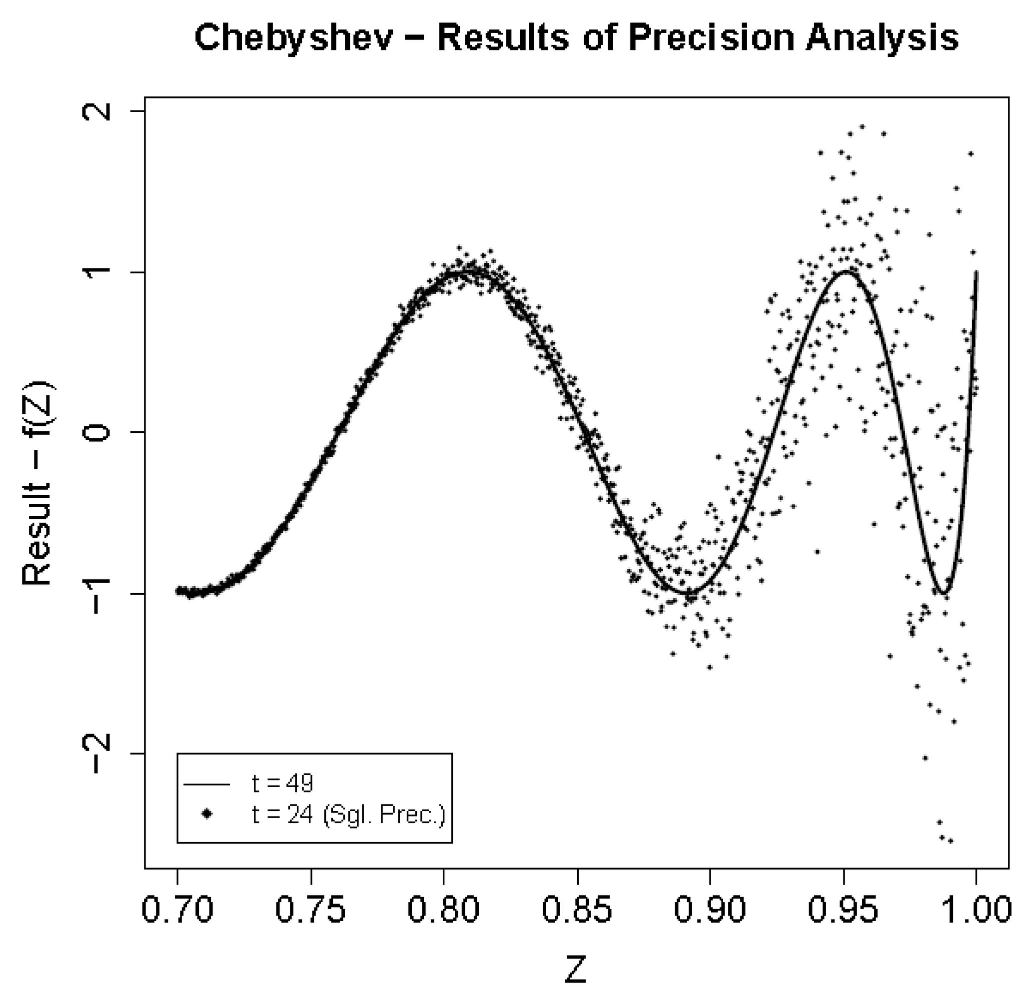

Figure 3: Optimization of Chebyshev Polynomial

CONCLUSIONS

- No automated methods for measuring sensitivity to rounding error available
- MCALIB – determine if single or double precision floating point arithmetic is required, compare implementations or optimize software for precision.
- First implementation of it's type to perform these functions using automated dynamic analysis methods.
- Represents a revolution in the field of error analysis.